Numerical Methods for PDEs

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Background:

This class can be thought of as Numerical Methods III. It follows on NM-II as taught by me or Leslie Greengard (Spring 2021), based on textbook

"FD methods for Ordinary & Partial Diff. Eqs" by R. LeVeque

which everyone should be comfortable with. Freely available as PDF so have it handy!

In particular, I will assume you are familiar with:

- Multi-step & multi-stage (Runge Kutta) schemes for ODEs including stability theory
- Basic finite-difference (FD) schemes for elliptic (Poisson), parabolic (heat) & hyperbolic (advection & wave) eqs., including von Neumann stability analysis (periodic regular grids).
- Basic spectral methods for PDEs, notably FFT-based (pseudo spectral) methods for periodic domains.

Homework 1 is posted on course webpage & helps me judge your background – please submit asap.

Also answer questions about yourself via email asap.

What I hope to cover:

① Finite Volume (FV) methods for conservation laws, focusing on advection-diffusion equations at first, then linear wave eq.'s and then nonlinear, including:

- artificial dispersion/dissipation (via modified eqs. & Fourier analysis)
- 2D & 3D

- boundary conditions
- limiters via reconstruction
- Godunov methods

(2) Finite element (FE) & Boundary Integral (BI) methods for elliptic PDEs from electrostatics (Poisson) & elasticity, including multigrid methods, variational formulations, and boundary conditions, via Galerkin methods.

(3) Fluid Dynamics, including

(a) MAC/FEM schemes for incompressible flow

(b) Riemann solvers for compressible flow (basics)

(c) Immersed boundary methods

(d) Boundary integral methods for Stokes flow

Grading

- Homeworks (especially in beginning to catch everyone up) graded by me with feedback.
- "Final" projects in second half, of your choosing with approval, focused on actual computation / codes / libraries or developing new methods.