Spring 2021: Numerical Analysis
Assignment 3 (due Wednesday March 17th 2pm)

1. **Induced matrix norms (5pts)**
   Let $A, B \in \mathbb{R}^{n \times n}$ and let the matrix norm $\|\cdot\|$ be induced by/subordinate of a vector norm $\|\cdot\|$.

   (a) (2pts) Show that $\|AB\| \leq \|A\|\|B\|$.

   (b) (1pt) For the identity matrix $I \in \mathbb{R}^{n \times n}$, show that $\|I\| = 1$.

   (c) (1pt) For A invertible, show that $\kappa(A) \geq 1$, where $\kappa(A)$ is the condition number of that matrix A corresponding to the norm $\|\cdot\|$. Use the above two properties with $B := A^{-1}$ for your argument.

   (d) (1pts) Argue that the Frobenius matrix norm $\|A\|_F := \left(\sum_{i,j=1}^{n} a_{ij}^2\right)^{1/2}$ cannot be induced by a suitable vector norm. *Hint*: Use one of the points above.

2. **QR factorization (3pts)**
   Compute by hand the QR-factorization of the matrix:

$$\begin{bmatrix} 9 & -6 \\ 12 & -8 \\ 0 & 20 \end{bmatrix}.$$

3. **Fitting (6pts)**
   We believe that a real number Y is approximately determined by X with the model function

$$Y = a\exp(X) + bX^2 + cX + d\ .$$

   We are given the following table of data for the values of X and Y:[1]

X	0.0	0.5	1.0	1.5	2.0	2.0	2.5
Y	0.0	0.20	0.27	0.30	0.32	0.35	0.27

   Using the above data points, write down 7 equations in the four unknowns a, b, c, d (2pts). The least squares solution to this system is the best fit function; find this in MATLAB/python/julia and explain which method you used and report the result (2pts). Plot the data points (X, Y) as points/symbols[2] and the best fit function as a smooth curve/line (2 pts).

[1] Note that you have two measurements at the same point $X = 2.0$. That is not uncommon in practice, and since measurements can contain noise it is possible that data at the same point are different.

[2] Do not connect the points; in MATLAB you can do that using `plot(X,Y,'ro')`.