Intro to NA

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$x = \sqrt{c}$ $\quad c > 0$

$x^2 = c$ nonlinear equation

$x \in \mathbb{R}$

$\sqrt{2} = 1.41 \ldots$?

$x = \mathrm{sqrt}(c)$

Babylonian method (iterative)

$$x_{k+1} = \frac{1}{2}\left(x_k + \frac{c}{x_k}\right)$$

index or iteration

$k = 0, 1, 2, 3, \ldots \infty$

(consistency)

(1) $\lim_{k \to \infty} x_k = \sqrt{c}$; $x_k \to \sqrt{c}$

Input : x_0 , $n_{iter} \in \mathbb{Z}^n$

k_{max}

Output : $x_{k_{max}} \simeq \sqrt{c}$

<u>An alternative:</u> Fixed-point method

$$x_{k+1} = p\,x_k + (1-p)\frac{c}{x_k}$$

$0 < p < 1$ why not $p=0,1$

$p=1/2$ is Babylonian

Is it <u>consistent</u>.

<u>If</u> it converges, then it converges to $\sqrt{c}$

$$x_k \xrightarrow[k\to\infty]{} \tilde{x}$$

$$x_{k+1} = \cancel{p\tilde{x}} + (1\cancel{-p})\frac{c}{\tilde{x}} = \tilde{x}$$

$$= \cancel{p\tilde{x}} + (1\cancel{-p})\tilde{x}$$

$$\frac{c}{\tilde{x}} = \tilde{x} \Rightarrow \tilde{x}^2 = c$$

$$\tilde{x} = \sqrt{c}$$

If $p = 1$

$x_{k+1} = x_k$ useless

If $p = 0$?

$$x_{k+1} = \frac{c}{x_k}$$

$$x_1 = \frac{c}{x_0} \qquad x_2 = \frac{c}{x_1} = x_0$$

Not converging

Is $p = 1/2$ better

yes, why?

Absolute error:

$$e_k = | x_k - \sqrt{c} |$$

Relative error

$$\varepsilon_k = \frac{| x_k - \sqrt{c} |}{| \sqrt{c} |}.$$