Worksheet 8 (April 21st, 2021)

1 TrueType-like fonts via splines

Spline-based interpolants are commonly used to represent shapes in computer graphics and engineering. In particular, one-dimensional quadratic and cubic Bézier curves (which are a variant on piecewise Hermite interpolation that uses geometrical control points to set the tangent vectors/derivatives) are used to represent letters in Type 1 (quadratic) and TrueType (cubic) fonts. Here you will use natural cubic spline interpolants to approximate letters and "render" (plot) them on the screen.

A curve in the plane can be represented as a parametric function of a real parameter $t \in [a, b]$, i.e., the points on the curve are $(x(t), y(t))$. The values of the parameter s are irrelevant so here it will simply be an integer index that labels points on the curve/letter.

1.1 Letter S

The table below gives sample values of $x(t)$ and $y(t)$ at several points along a curve representing the letter S. Render this letter (i.e., plot the letter) by interpolating $x(t)$ and $y(t)$ using a piecewise cubic natural spline interpolant.

t	0	1	2	3	4	5	6
x	1	2	3	2	1.2	2	2.7
y	1	0	1	2.5	3.4	4	3.2

Note: While this is not how actual font rendering works these days, it illustrates the idea. In particular, note that simply multiplying $x(t)$ and $y(t)$ by a scaling factor s will scale the letter (set the font size), which is a crucial property of modern fonts and graphics and is different from traditional pixel-based or rasterized representations of shapes/curves/letters.

1.2 Mystery letter

Now repeat the same for the letter given below, reusing as much of the code as possible.

t	0	1	2	3	4	5	6	7	8	9	10	11
x	3	1.75	0.90	0	0.50	1.50	3.25	4.25	4.25	3	3.75	6.00
y	4	1.60	0.50	0	1.00	0.50	0.50	2.25	4.00	4	3.25	4.25

1.3 Difficult letters

Create a table and render a letter O and a letter Z on your own as a group. Comment on any difficulties you encountered and how you may be (or were) able to avoid them.