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## I. INTRODUCTION

Diffusion of colloidal particles confined to two-dimensional surfaces is a key transport mechanism in several contexts of technological and biological significance. Colloidal particles can spontaneously absorb on fluid-fluid interfaces and stabilize Pickering emulsions [1]. The transverse diffusion of proteins embedded in lipid bilayers controls their biological function [2]. In man-made colloidal suspensions, colloidal particles can be confined to primarily diffuse in a plane by walls [3] or electrostatic forces [4]. While much is understood about complex fluid-fluid interfaces [1], fundamental questions about diffusive transport at interfaces remain unanswered [5].

Bulk diffusion of particles in liquids is well-known to be controlled by hydrodynamics, and diffusion on interfaces is no exception. While the diffusion of colloids and polymers on a fluid-fluid interface has been studied theoretically since the 1970s [6, 7], *collective diffusion* in a monolayer of colloidal particles confined to a fluid-fluid interface has only recently been explored in some detail [8–10]. These recent studies have shown that collective diffusion on interfaces is anomalous, with the short-time collective diffusion coefficient diverging as the inverse of the